Swerve Drive Kinematics

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1 Introduction

The goal of this kinematic analysis is to describe the motion of a Wheeled Mobile Robot (WMR) with a swerve drive locomotion system. Inverse kinematics is used to determine the what control inputs are required to achieve the desired motion of the robot. Forward kinematics is used to determine what motion will occur with the current control inputs. The use of both inverse and forward kinematics allows for the computation of an error signal, which can then be used to drive a PID controller for precise control of robot.

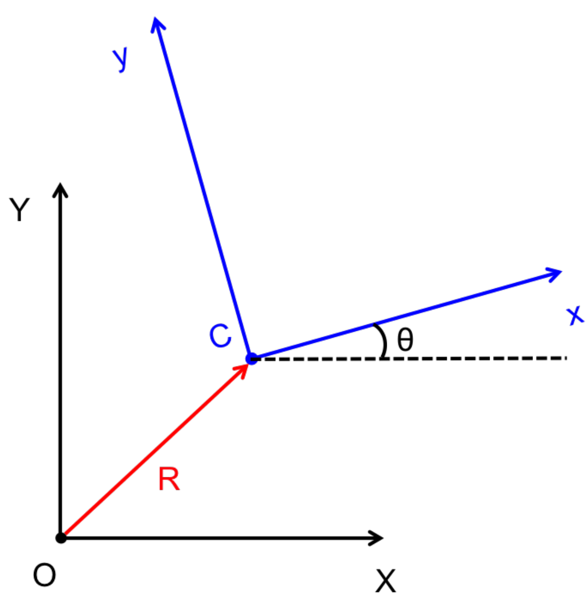

Figure 1: Reference frames OXY and Cxy

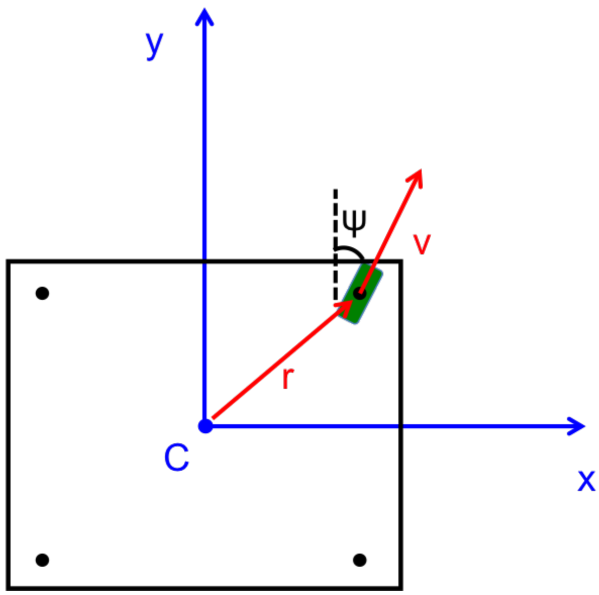

Figure 2: Reference frame Cxy, with robot overlaid. Wheel angle ϕ also shown.

2 Symbols

- OXY: Inertial or ground reference frame.
  - $\hat{I}$: Unit vector corresponding to $+X$ direction.
  - $\hat{J}$: Unit vector corresponding to $+Y$ direction.
  - $\hat{K}$: Unit vector corresponding to $+Z$ direction.
- Cxy: Robot reference frame. Origin is the centroid of the rectangle formed by the wheel locations.
  - $\hat{i}$: Unit vector corresponding to $+x$ direction.
  - $\hat{j}$: Unit vector corresponding to $+y$ direction.
  - $\hat{k}$: Unit vector corresponding to $+z$ direction.
- $\vec{R}$: Position vector extending from the origin of OXY to the origin of Cxy.
- $\vec{V}_{Cxy \to OXY} = \frac{d\vec{R}}{dt}$: Velocity of the robot frame Cxy with respect to the inertial reference frame, OXY.
- θ: Angle from the X axis of the OXY frame to the x axis of the Cxy frame.
- $\vec{\Omega}_{Cxy \to OXY} = \frac{d\theta}{dt}$: Angular velocity of the robot frame Cxy with respect to the inertial reference frame, OXY.
- $\vec{v}^{i}_{Cxy \to OXY}$: Velocities of each of the four wheels with respect to the ground frame.

- $\vec{v}^i_{rot}$: Component of wheel velocity contributing to angular velocity Ω of the robot.
- $\vec{v}^i_{lin}$: Component of wheel velocity contributing to linear velocity of V of the robot.
- ψ^i: Angle of the wheels with respect to the robot frame.
- ω^i: Angular velocity of each of the four wheels.
- $\vec{r}^i_{Cxy}$: Position vector of each wheel in the robot frame Cxy.
- b: Wheelbase of the robot.
- w: Track width of the robot.

3 Inverse Kinematics

First, use a standard coordinate frame transformation to obtain the velocities of the wheels with respect to the ground reference frame.

$$\vec{V}_{Cxy\to OXY} = \frac{d\vec{R}}{dt} = V_X\hat{I} + V_Y\hat{J} \tag{1}$$
$$\vec{\Omega}_{Cxy\to OXY} = \Omega\hat{K} \tag{2}$$
$$\vec{r}^i = r^i_x\hat{i} + r^i_y\hat{j} \tag{3}$$
$$\vec{v}^i_{Cxy\to OXY} = \frac{d\vec{R}}{dt} + \vec{\Omega}\times\vec{r}^i \tag{4}$$
$$= V_X\hat{I} + V_Y\hat{J} + \Omega\hat{K}\times(r^i_x\hat{i} + r^i_y\hat{j}) \tag{5}$$
$$= V_X\hat{I} + V_Y\hat{J} + r^i_x\Omega(\hat{K}\times\hat{i}) + r^i_y\Omega(\hat{K}\times\hat{j}) \tag{6}$$
$$\hat{K} = \hat{k} \tag{7}$$
$$\hat{K}\times\hat{i} = \hat{j}; \hat{K}\times\hat{j} = -\hat{i} \tag{8}$$
$$\vec{v}^i_{Cxy\to OXY} = V_X\hat{I} + V_Y\hat{J} + r^i_x\Omega\hat{j} - r^i_y\Omega\hat{i} \tag{9}$$
$$\hat{i} = \hat{I}\cos(\theta); \hat{j} = \hat{J}\cos(\theta) \tag{10}$$
$$\vec{v}^i_{Cxy\to OXY} = V_X\hat{I} + V_Y\hat{J} + r^i_x\Omega\cos(\theta)\hat{J} - r^i_y\Omega\cos(\theta)\hat{I} \tag{11}$$
$$\vec{v}^i_{Cxy\to OXY} = \langle V_X - r^i_y\Omega\cos(\theta), V_Y + r^i_x\Omega\cos(\theta)\rangle \tag{12}$$

Looking at the resulting equation, we see that the velocity of the wheels is simply the linear velocity of the centroid of the robot plus the velocity required to rotate the robot. Notice that the rotational component is dependent on the angle of the robot in the ground reference frame. This is because as the robot rotates, the relative velocity of the wheels must change. An analogous example is one of a bicycle wheel; the relative velocity on the top of the wheel is greater than the relative velocity of the center of the wheel. You can imagine that if you were sitting on the wheel, you would experience a oscillatory motion, as you sped up towards the top of the wheel, then slowed to zero at the bottom of the wheel.

A more difficult result to explain is the negative sign in the X component of the velocity. This can be interpreted as a consequence of the sign convention selected for the problem.

From these wheel velocities, the rotation speeds and angles of the wheels can be easily defined with basic geometry.

$$\omega^i = \frac{|\vec{v}^i|}{r_{wheel}} \tag{13}$$

$$\psi^i_{Cxy} = \arctan\left(\frac{|\vec{v}^i \cdot \hat{j}|}{|\vec{v}^i \cdot \hat{i}|}\right) \tag{14}$$

$$\psi^i_{Cxy} = \arctan\left(\frac{|\vec{v}^i \cdot \hat{J}\cos(\theta)|}{|\vec{v}^i \cdot \hat{I}\cos(\theta)}\right) \tag{15}$$

The rotational speed of the wheel is simply the magnitude of the velocity divided by the radius of the wheel. The angle of the wheel is the direction of the velocity vector. The angle of the wheel with respect to the body of the robot is desired, since the angles will be measured from the robot. The steering angle is still dependent on the angle of the robot to the inertial reference frame. This is shown when converting the unit vectors from the robot frame to the unit vectors of the inertial frame.

For a standard four wheeled robot, the position vectors for the wheels can be defined as:

$$r_1 = \langle -w/2, b/2 \rangle \tag{16}$$

$$r_2 = \langle w/2, b/2 \rangle \tag{17}$$

$$r_3 = \langle w/2, -b/2 \rangle \tag{18}$$

$$r_4 = \langle -w/2, -b/2 \rangle \tag{19}$$